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Vol 36 Nº2 2021   www.ricuc.cl

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Large deformations in barrel vaults: from anti-funicular to funicular vaults

Grandes deformaciones en bóvedas de cañón: de formas antifuniculares a bóvedas funiculares

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Abstract

In the Aran Valley there is a group of Romanesque churches built between the 11· and 13· centuries characterised by their large deformations, in some cases exceeding 7% of their original condition. The research shows the results of the geometric study carried out with a Terrestrial Laser Scanner (TLS), revealing geometric differences with regard to the traditional definition of barrel vault. The deformations of the initially anti-funicular vaults are sometimes transformed into convex funicular shapes, where the final spatial volume is smaller than the initial one, despite the fact that the linear length of the curve and surface of the vault is greater than that of their original condition.

Keywords: Masonry, vaults, funicular, anti-funicular, limit analysis

Resumen

En la Val d'Aran existe un conjunto de iglesias románicas construidas entre los siglos XI y XIII caracterizadas por sus grandes deformaciones, en algunos casos superior al 7% de su condición inicial. La investigación muestra los resultados del estudio geométrico realizado con Escáner Láser Terrestre (TLS), revelando diferencias geométricas con la tradicional definición de bóveda de cañón. Las deformaciones de las bóvedas inicialmente antifuniculares se convierten en algunos casos en formas funiculares convexas, donde se cumple que el volumen espacial final generado es menor que el inicial, pese a que su desarrollo lineal de la curvatura y la superficie es mayor que la de su estado primitivo.

Palabras clave: *Obra de fábrica, bóvedas, funicular, antifunicular, análisis límite*

1. Introduction

In the region of the Aran Valley (Spain), located in the northern gradient of the Pyrenees bordering the Department of Haute Garonne (France), there are urban settlements established around the course of the Garonne River, where Romanic churches were built between the 12th and 13th centuries. These churches have suffered large deformations. They drew the attention of Eugène-Emmanuel Viollet-le-Duc during his visit to Bossost in 1883 (Español, 2013), Lluís Domènech i Montaner, who visited them with the School of Architecture of Barcelona in 1905 (Granell, 2006), Josep Puig i Cadafalch, with the historical-archeological expedition of the Institut d'Estudis Catalans in 1907, (Puig i Cadafalch, 1908), or Juan Bassegoda Nonell, who suggests that these formal anomalies are one of the characteristics of the Romanesque architecture in Catalonia (Bassegoda Nonell, 1974) (Figure 1).

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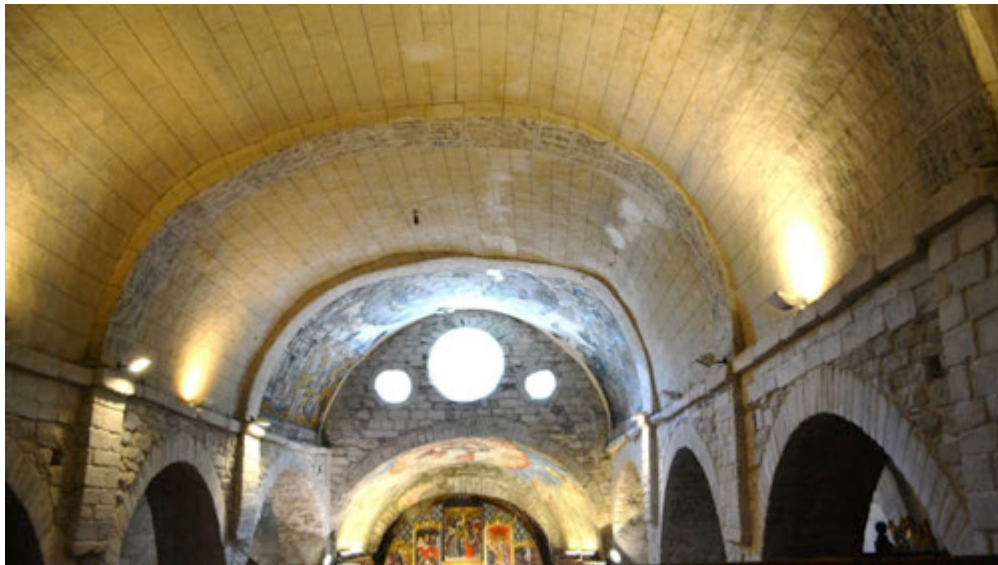
Figure 1. Masonry structure with large deformations, Santa Maria d'Arties

        The objective of the study is to synthesize the theoretical framework, based on the analytic geometry of the vaults that can adopt convex shapes after suffering large deformations; therefore, data was captured with terrestrial laser scanner (TLS) during the period of (2015-2020). These studies have allowed calibrating the deformations and displacements in the leaning of walls and pillars and in the formation of the articulations of the vault, which are responsible of the transformation from anti-funicular to funicular shapes. This is the case of the churches of Santa Eularia d'Unha (12th c.), Santa Maria d'Arties (12th-13th c.) and Era Purificació de Bossost (12th-13th c.) (Figure 2). In the case of Santa Maria d'Arties, consolidation interventions were carried out by Joan Bassegoda i Nonell in 1975 (Sáez, 1976)], and Joan Josep Polo i Berroy in 2009 (Polo and Cots, 2009).

Figure 2. Convex funicular vaults in Santa Eularia d'Unha (12th c.), Santa Maria d'Arties (12th-13th) and Era Purificacio de Bossost (12th-13th c.)

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The methodology starts with a 3D point cloud surveyed with the (TLS) Leica ScanStation P20, with an accuracy of 3 mm at 50 m, thereby obtaining a point cloud with the Cyclone software, with an error of alignment of 3 mm, generating a 3D mesh with a triangle measure of 2.5 cm with the 3DReshaper software. The verification of these deformed arches adheres to the limit analysis theory of Jacques Heyman, who states that, if the arches of these shapes remain standing, it is because a catenary shape can be drawn inside the arch, whose formula is: $(x_c) = m \cdot \cos h\left(\frac{x}{m}\right)$ (Heyman, 1999).

The status of the issue of the barrel vault was brought up by Eugene-Emmanuel Viollet-le-Duc in his Dictionnaire raisonné de l'architecture (1854 - 1868) (Viollet-le-Duc 1842) estimating the interaction between the Romanesque vault and the wall. August Choisy also referred to them in the Roman construction (Choisy, 1873) and the Byzantine construction (Choisy, 1883). Likewise, Josep Puig i Cadafalch, in the Historia General del Arte (1901), nuances the difference among Romanesque vaults, where the elasticity replaces the stability of the more monolithic Roman ones. In his L'Arquitectura Romànica a Catalunya (1918) he outlines the special characterization of this masonry constructions.

Bonaventura Bassegoda i Musté (Bassegoda Musté, 1944) approaches it from the perspective of the elastic theory, and Leopoldo Torres Balbás (1946) addresses this issue from the typological point of view in Bóvedas romanas sobre arcos de resalto. In addition, Juan Bassegoda Nonell considers that traditional Roman models gave these medieval vaults a structural shape (Bassegoda Nonell, 1977).

2. General Assessment of the Vaults Equilibrium

The case study of Santa Maria d'Arties is used for verifying the limit state dealing with the equilibrium of these vaults, thereby defining two sections, one (T_{p1}) located between the spans [V_1-V_2], and the other corresponding to the lateral nave with funicular vault (T_{v2}) (Figure 3). The first hypothesis, related to the initial undeformed section, reveals that the structural design allows multiple states of equilibrium, where several thrust lines in the section can be defined (Figure 4.a). The second one, referring to the structural analysis of the most deformed section (T_{p1}), whose different combinatorial analyses reveal that there are only a few thrust lines in the section, thus fulfilling the principles of equilibrium with a small safety margin (Figure 4.b).
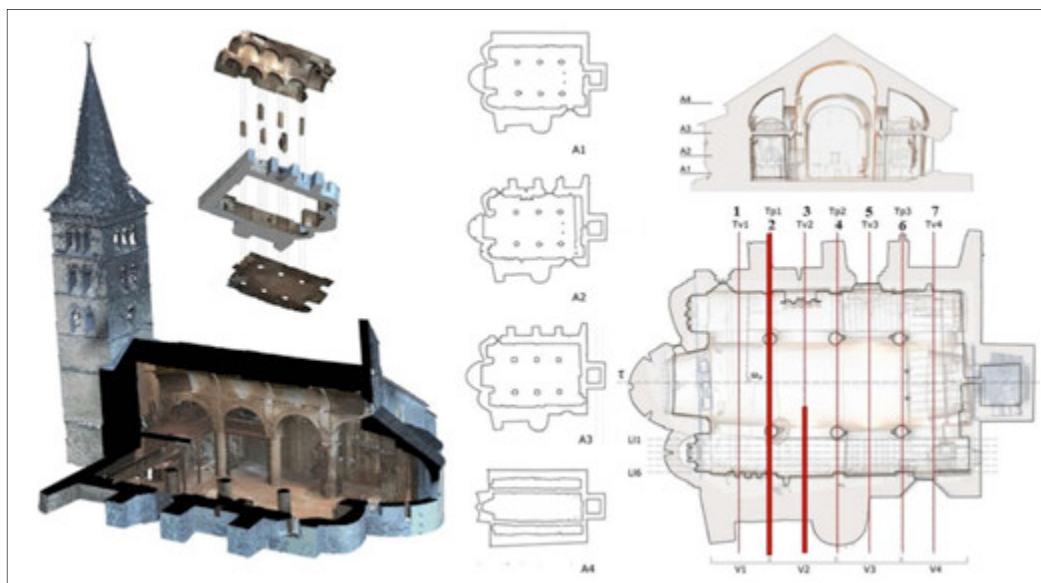

Figure 3. Santa Maria d'Arties. Methodology and definition of the calculated sections

On the other hand, the analysis of the section of the vaults' funicular shapes is verified (T_{v2}), which supposes a particular case in the context of the overall structure. The graphical verification shows that, despite the funicular deformations and according to its maximum thrust, it is possible to find a solution contained in the masonry section in the hypotheses of the thickness interval of [0.50-0.25 m] (Figure 4.c).
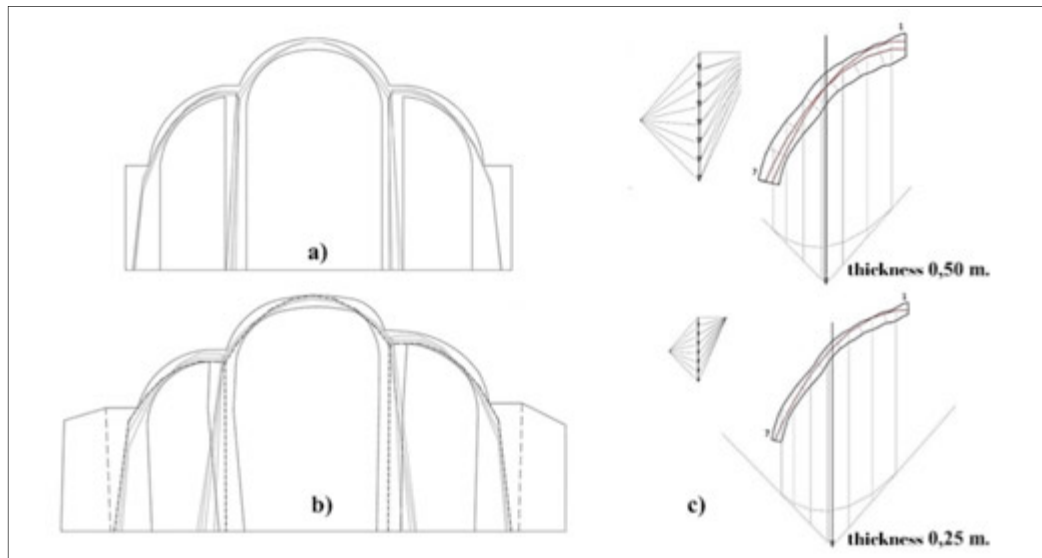

Figure 4. *Verification of the limit analysis in the section; a,b) (T_{p1}); c) (T_{v2})*

From the mechanical point of view concerning the current condition of the vault, it is possible to draw a thrust line contained in its section, and in the wall section; therefore, the interaction of the buttress would be unnecessary. In the verification of the quarter-sphere vaults, the results have evidenced that, despite the identified funicular shapes, it is still possible to draw a thrust line in that section. The construction of these abutments responds to the need to stabilize the collapse mechanism of the structure.

This deformation capacity shows the relevance of the stiffness of masonry regarding the stereotomy and size of the masonry as well as the mechanical properties and mortar joints, which varies in the different Aran Valley churches (Figure 5). Consequently, the results of the analysis evidence a geometric irregularity and huge deformability of the masonry without reaching the point of collapse.

Figure 5. *Relationship between m² of masonry and mortar in Aran Valley churches*

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3. General Assessment of the Vaults Deformations

In general terms, Romanesque architecture is built with half dome or quarter-sphere vaults, so when they present a regular geometry and a monolithic construction, the thrust is estimated as perpendicular to its directrix. The thrust E_b (E_{bx}, E_{by}, E_{bz}) and its vectorization E_b (0, E_{by}, 0), assumes homogeneous vaults and constant directrix as indicated by August Choisy (Choisy, 1873). The obtained geometric data demonstrate that the floor plan generates conical vaults, as theorized by Joan Bassegoda (Bassegoda, 1974), the stereotomy is not homogeneous but rather exhibits an irregular distribution, as established by Josep Puig i Cadafalch (Puig i Cadafalch, 1901). In turn, Josep Puig i Cadafalch (Puig i Cadafalch, 1908) indicates that there is a large difference in the stiffness between the vaults' supports, pillars and walls, a fact that was also observed by Luis Villanueva in the quarter-sphere vaults with funicular shapes (Villanueva, 1974).
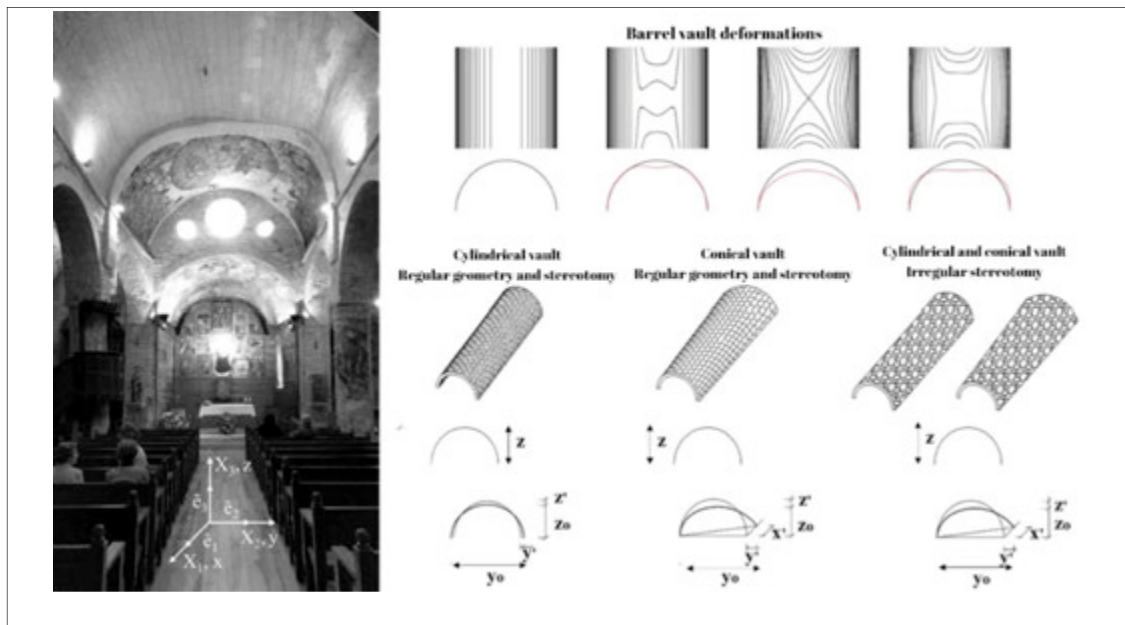

Figure 6. *Characterization of the vaults deformations*

These conditions diverge completely from the general thrust theory of homogenous vaults with cylindrical directrix E_b (0, E_{by}, 0). In any of the previous hypotheses, or combination thereof, thrusts act in at least two directions (x, y), and the resultant will not be perpendicular to the vault directrix E_b (E_{bx}, E_{by}, 0), and there is settlement at the imposts E_b (E_{bx}, E_{by}, E_{bz}) (Figure 6).

A continuum model formed by material points occupying different positions in the space during their movement in a period [Ω_0- Ω_t], can be simulated with a Cartesian coordinate system (X,Y,Z) with orthonormal basis ($ê1'$, $ê2$, $ê3$) referred to Ω_0, with a vector X, which initially occupied a point P in the space defined by material coordinates: X=X1 $ê1$+ X2 $ê2$+ X3 $ê3$= Xi êi; (X1, X2, X3). In the current deformed configuration Ω_t, the particle originally located in P occupies the spatial point P´ and its position vector x=x1 $ê1$+ x2 $ê2$+ x3 $ê3$= xi êi; (x1 x2 x3) in spatial coordinates in time t (Figure 7).

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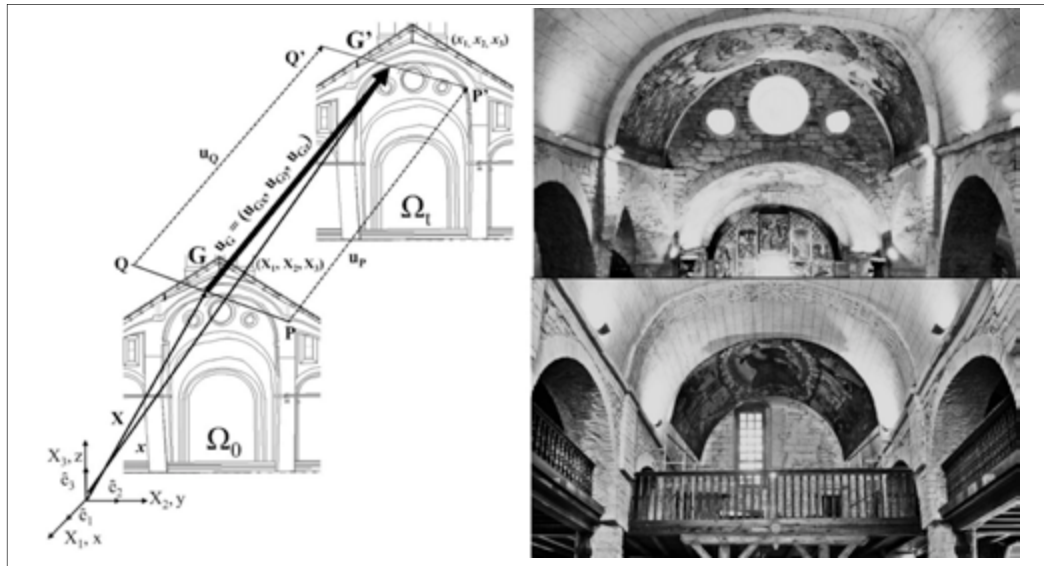
Figure 7. Characterization of vault displacements in Santa Maria d'Arties

The vectorization establishes that the thrusts Eb (Ebx, Eby, Ebz) cause displacements in the three directions, df (df$_x$, df$_y$, df$_z$). The vaults of the central nave are supported by the walls of the main arches, which in turn are supported by the pillars presenting deformations df$_p$(df$_{px}$, df$_{py}$, df$_{pz}$). The vaults of the lateral naves are supported by the exterior walls df$_m$(df$_{mx}$, df$_{my}$, df$_{mz}$).

Consequently, the deformations and displacements are directly related to the stereotomy of the pillar based on the thrust E$_p$ (E$_{px}$, E$_{py}$, E$_{pz}$). In some cases, there can be differential settlements in direction (z), which causes (E$_{pz2}$); otherwise, in direction (x,y), given the effects of the internal distribution of the masonry (E$_{pxy1}$) and the irregular geometry of the vaults (E$_{pxy2}$), these settlements may tend to even out (E$_{pxy1}$ - E$_{pxy2}$) or quite the opposite (E$_{pz1}$+ E$_{pz2}$). The pillars are built with discontinuous materials and a large number of joints, so the upper part (Δx, Δy) has a general tendency to move, and if not, differential settlements (Δz=0) are produced (Figure 8).

Figure 8. Pillars a) Santa Maria de Cap d'Aràn de Tredós; b) Sant Félix de Vilàc ; c) Era Purification de Bossost

The deformations produced are of the type df$_p$ (df$_{px}$, df$_{py}$, 0). One way of calculating the pillar displacement is to analyze the centroid of the pillar joint through its n sections (n$_s$) using the coordinates (x$_{ci}$, y$_{ci}$, z$_{ci}$) (Figure 9).

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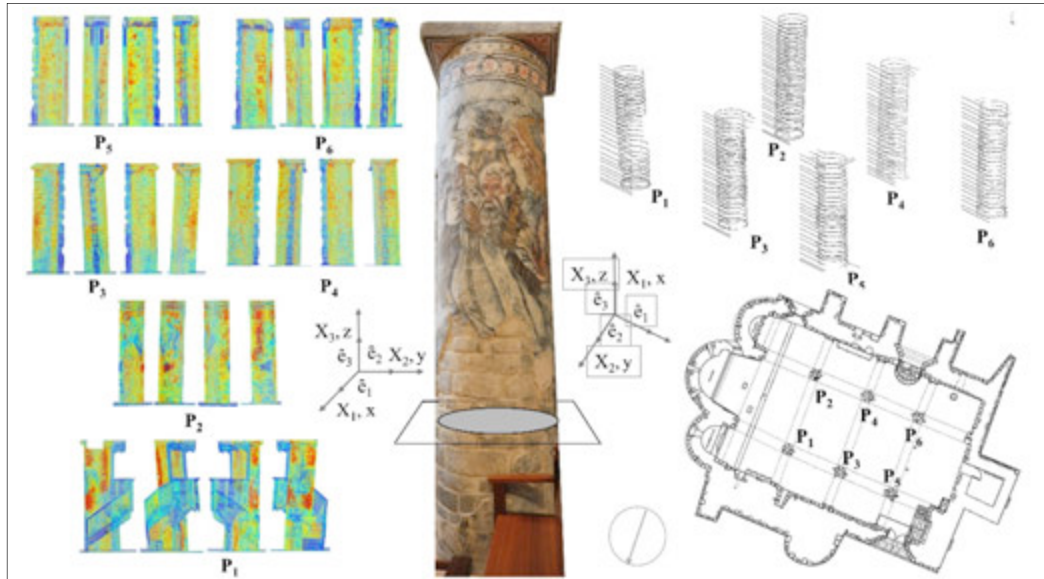

Figure 9. *Characterization of the pillar displacements in Santa Maria d'Arties*

This allows calculating a point matrix starting from the initial centroid of the non-deformable floor plan (x_{c0}, y_{c0}, z_{c0}) to that located on the impost (x_{cs}, y_{cs}, z_{cs}). These points define a function f (c_i) and enable calculating the tendency to a regression plane (τ), which will characterize the approximation to a displacement vector of the structural span acting on the pillar, defining the clearance angle (ω) of the plane (τ) (Figure 10).

a)   (ω_τ) = 90°, perpendicular to the directrix φ_1
b)   (ω_τ) < 90°, displacement towards the apse
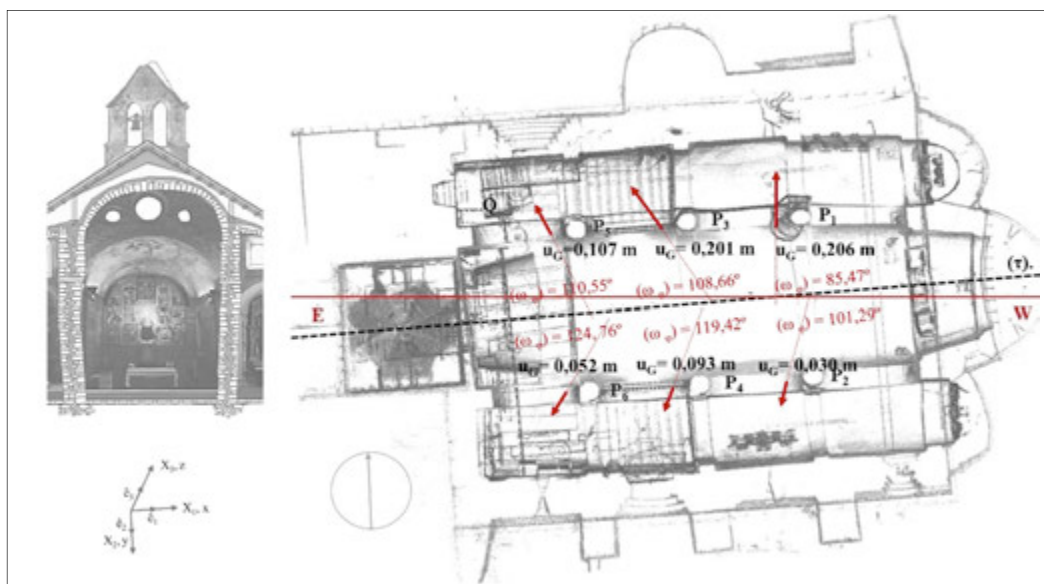c)   (ω_τ) > 90°, displacement towards the façade.

Figure 10. *Characterization of the regression plane of pillar displacements*

The walls have a higher stiffness than the pillars, and tend to re-equilibrate the stresses that the thrust of the vaults of lateral naves E_b (E_{bx}, E_{by}, E_{bz}) exercises on the wall E_m (E_{mx}, E_{my}, E_{mz}). This action develops along the entire length of the wall, and the critical points are found in both extremities. The latter tend to be equilibrated by the apsidioles' curved walls or by locating the bell towers on the opposite façade.

In some of them, a construction system is provided that can hide, in the interior of the wall, a timber beam that absorbs the tensile stresses of horizontal thrusts, a solution that is visible and used in certain apses (Figure 11).

Figure 11. Timber tie beams a) Santa Maria de Cap d'Aràn de Tredós; b) Sant Joan d'Arties

The deformation due to the horizontal thrust (E_{mx}, E_{my}) causes leaning because the masonry lacks stiffness, and if the supports have not suffered any differential settlements ($\Delta z=0$), the deformations are df_m (df_{mx}, df_{my}, 0), or if they have, df_m (df_{mx}, df_{my}, df_{mz}). The walls' deformations df_m (df_{mx}, df_{my}, df_{mz}) due to the vault thrusts E_m (E_{mx}, E_{my}, E_{mz}) cause the displacement of the inner face of the inner leaf in contact with the quarter-sphere vault with a deformation (df_{mx}, df_{my}), thereby transmitting it to the external leaf. A buttress system has been provided through the construction of large counterforts to contain these deformations (Figure 12).

Figure 12. Counterforts of Santa Maria d'Arties, which produce passive thrust (E_{pm})

4. Evolution/Transformation From Concave to Convex Vaults

The vaults' active thrust (E_{ba}) on the pillars and walls has been calculated. However, in order to understand the equilibrium of these constructions, the role of the buttressing system is essential because it exerts the passive thrust of the abutment elements (E_{pm}), thus causing, in some cases, the convexity of these vaults, which have presented funicular shapes (Figure 13).

The analysis of these shapes can only be understood through a 3D analysis with an interval [a, b], which has to impose the equilibrium condition by means of the elastic theory and, therefore, the masonry of the vaults works mechanically under compressive strength.

$$\sum_{b}^{a} F_{(x,y,z)}\left(E_{ba} + E_{pm}\right)=0 \text{ y}$$
$$\sum_{b}^{a} M_{(x,y,z)}\left(E_{ba} + E_{pm}\right)=0$$

Figure 13. Funicular vault of Santa Maria d'Arties

The study of the deformations of construction elements caused by the geometry of the vault of the depressed barrel of the central nave and that of the pointed lateral ones, causes the action of the own weight, and the external ones (snow, wind and earthquakes) together with the mortar loss in the masonry joints due to humidity and vibrations, to bring the primitive function of the vault $fi_{(x,y,z)}$ to deform towards a function $ff_{(x,y,z)}$ (Figure 14).
The observation and analysis allow comparing the linear, two-dimensional and three-dimensional conditions of the primitive function $fi_{(x,y,z)}$ and the deformed $ff_{(x,y,z)}$ assessed in a time in the interval [Ω_0 - Ω_t]. As a general criterion, it was established that, given a vault, the length of the section of the deformed vault $ff_{(x)}$ is higher or equal than the primitive function $fi_{(x)}$.

$$\int_{yb}^{ya} f dy \geq \int_{yb}^{ya} fi dy \qquad\qquad (1)$$

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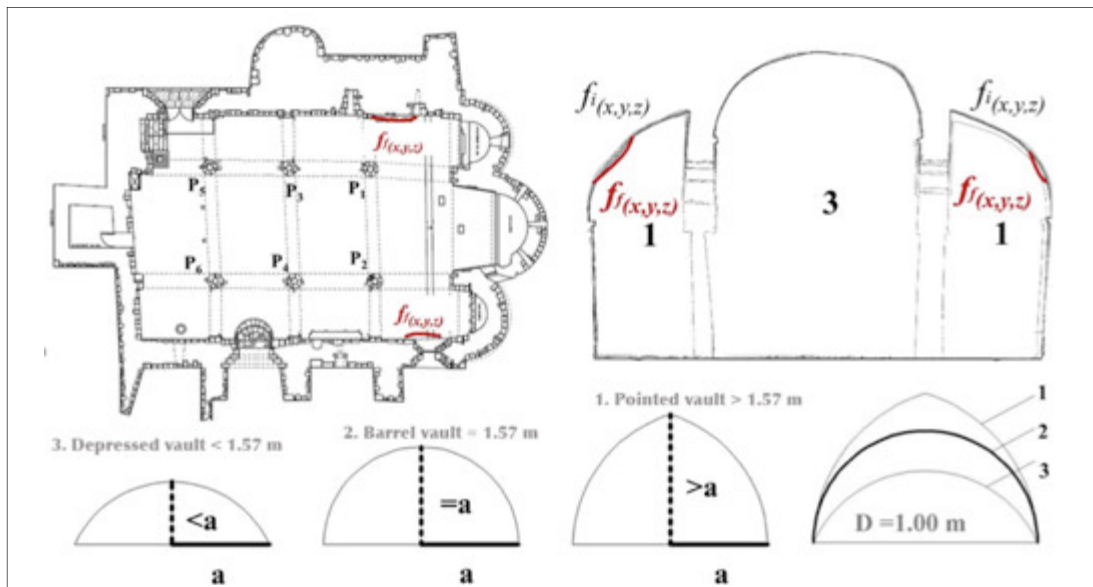

Figure 14. *Development of the section of the vaults of Santa Maria d'Arties*

That the surface of the section of the deformed vault $ff_{(x,y)}$ is greater or equal than that of the primitive function $fi_{(y,z)}$

$$\iint_{yzb}^{yza} ff\,dy\,z \geq \iint_{yzb}^{yza} fi\,dyz \tag{2}$$

Finally, we consider that the volume of the interior interval [a, b] of a vault $ff_{(x,y,z)}$ can be equal, larger or smaller than that of the primitive function $fi_{(x,y,z)}$. Therefore, if the volume of the interior space generated by a vault is greater or equal than the initial one, its curvature is convex and anti-funicular, so that

$$\iiint_{xyzb}^{xyza} ff\,dxyz \geq \iiint_{xyzb}^{xyza} fi\,dxyz \tag{3}$$

On the other hand, if the volume of the deformed vault is smaller than the initial form, the vault is funicular and concave, so that

$$\iiint_{xyzb}^{xyza} ff\,dxyz < \iiint_{xyzb}^{xyza} fi\,dxyz \tag{4}$$

We can also define this concept in terms of the concavity or convexity of the function of the deformed $ff_{(y,z)}$ on the interval [a, b] on a plane perpendicular to that of the vault directrix φ_1. So, if it is concave, the function is funicular, so that

$$ff''_{(y,z)} < 0 \tag{5}$$

While, if the deformed vault is convex and anti-funicular

$$ff''_{(y,z)} > 0 \tag{6}$$

Another characteristic that defines the function is the deformed $ff_{(x)}$, the maximum point will be the one having a horizontal tangent

$$ff''_{(y,z)} = 0 \tag{7}$$

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   The study of the deformations defines the behavior of the building considering its history on the interval [Ω0 - Ωt], and the evolution of different interventions, such as the construction of counterforts and the location of the bell towers. These actions have conditioned the active thrusts of the vaults (E_{ba}), which have gradually activated a kinematic mechanism on the original construction. These elements have operated as vertical counterforts, producing passive thrusts (E_{pm}), thus tending towards the condition of equilibrium of the elastic theory.

   The vaults of the central naves are much more lowered than the half barrel ones located in the side aisles. Furthermore, the geometry of the floor plan has a conical directrix, whose transverse axis of the plane (τ) tends to behave as a beam supported on both extremes, where the largest deformation occurs in the two central vanes (Figure 15). The extremes of these buildings are essential for the overall stability of the construction. The apses and façades with bell towers are stiffer elements and the near vaults tend to have smaller deformations.
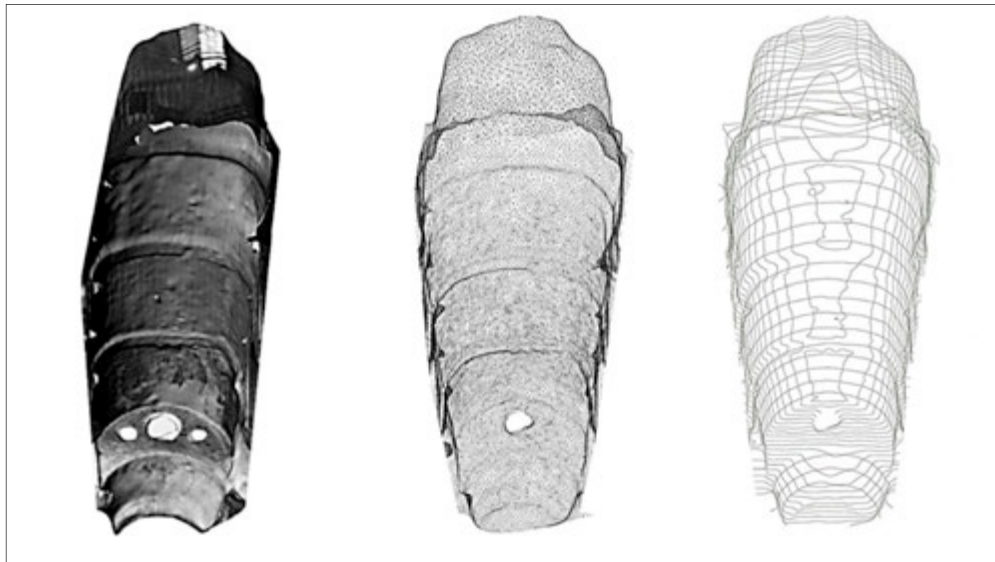

Figure 15. *Topography of barrel vaults in Santa Maria d'Arties*

   Regarding the apse, the curvature of the walls defines its stiffnes, but that is not the case with the façade, where the bell towers were built centuries later to serve as counterfort elements (Figure 16).

Figure 16. *Bell towers of Bagergue, Salardú and Vilac*

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5. Conclusions

The research allows to conclude that the geometry of the built model without deformations should not have presented equilibrium issues; therefore, the building's deformations are not due to a design problem. The typology of identified displacements reveals that there are not many foundation problems and, consequently, the deformations of the structure are associated to the mortar's poor mechanical properties, together with its irregular and small-sized masonry. Thus, a stiffness rather than a stability problem is identified, which explains why the structure has gradually settled over time, due to the deterioration of the mortar and the ratio mortar/stone of the masonry.

Once this process began, the eccentricity of the loads on the system and the active thrusts of the vaults (E_{ba}) activated a kinematic mechanism on the original section, which was sought to be counteracted by passive thrusts (E_{pm}) with large buttresses (Figure 12). The vaults, which initial shape were anti-funicular, have ended up creating funicular shapes, which confirms the hypothesis that the final spatial volume is smaller than the initial one, despite the fact that the linear development of the curvature and the surface are greater than those of their original condition.

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